Cowboy probability

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Suppose there are 3 cowboys A , B, C . They get into an argument and decide to settle it with a "three-way duel" with Cowboy A shoots first , possibly killing one of the other two. Cowboy B, if he is still alive, will then take a shot . Cowboy C will then take a shot if he is still alive. After that, Cowboy A will take the turn again, and so on. The turn goes round and round until only one cowboy remains.

Cowboy A can hit his target with a probability of 1/3 , Cowboy B with a probability of 1/2 and Cowboy C is a good shot and can hit his target every time, that is, with a probability of 1.

If **you** are Cowboy A , what can you do to maximize your chance of survival ?

Hint 1

You can choose among the three possible ways :

(I) You shoot at Cowboy B .

(II) You shoot at Cowboy C.

(III) You shoot at the air.

Hint 2

You may investigate "two-way duel" first .

Solution You are Cowboy A. Let us investigate "two-way duel" first .

(1) Suppose you are in a "two-way duel" with Cowboy C.

(a) Probability that you shoot first and you survive $= P_1 = \underline{\underline{\frac{1}{3}}}$

(b) Probability that C shoots first and you survive $= P_2 = \underline{\underline{0}}$

(2) Suppose you are in a "two-way duel" with Cowboy B.

(a) Probability that you shoot first and you survive $= P_3$

= P(you hit B in 1st shot) + P(you miss in 1st shot, B misses in 2nd shot, you hit B in 3rd shot) + …

$= \frac{1}{3}+\frac{2}{3}\times\frac{1}{2}\times\frac{1}{3}+\frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}\times\frac{1}{3}+...$, which is an infinite geom. series, common ratio $= \frac{2}{3}\times\frac{1}{2}$

$= \frac{1/3}{1-1/3} = \underline{\underline{\frac{1}{2}}}$ (use the formula: $S(\infty) = \frac{a}{1-r}$)

(b) If Cowboy B shoots first and he misses you, what remains is a duel with Cowboy B and you shoot first.

Probability that Cowboy B goes first and you survives $= P_4$

$= \frac{1}{2}\times P_3 = \frac{1}{2}\times\frac{1}{2} = \underline{\underline{\frac{1}{4}}}$

Now, we come back to investigate the "three-way" duel.

(I) Suppose you shoot at Cowboy B .

(a) If you hit Cowboy B and he dies, then it is Cowboy C's turn and your chance of survival is $P_5 = 0$.

(b) If you miss Cowboy B and Cowboy B must shoot at Cowboy C to maximize his chance of survival.

(i) If Cowboy B hits Cowboy C, then it becomes a "two-way duel" between you and Cowboy B with you goes first.

Therefore your probability of survival is $P_6 = \frac{2}{3} \times \frac{1}{2} \times P_3 = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{6}$

(ii) If Cowboy B misses in shooting Cowboy C, then it is Cowboy C turn and he kills Cowboy B to maximize his chance of survival.

It becomes a "two-way duel" between you and Cowboy C with you shoot first.

Therefore your chance of survival is $P_7 = \frac{2}{3} \times \frac{1}{2} \times P_1 = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{9}$

$\therefore$ Total probability of your survival = $P_5 + P_6 + P_7 = 0 + \frac{1}{6} + \frac{1}{9} = \underline{\underline{\frac{5}{18}}}$

(II) Suppose you shoot at Cowboy C .

(a) If you hit Cowboy C and he dies, then it becomes "two-way duel" between you and Cowboy B with Cowboy B shoots first, and your chance of survival is $P_8 = \frac{1}{3} \times P_4 = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

(b) If you miss Cowboy C and Cowboy C must shoot at Cowboy B and kills Cowboy B.

It becomes is a "two-way duel" between you and Cowboy C with you shoot first.

Your chance of survival = $P_9 = \frac{2}{3} \times P_1 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

$\therefore$ Total probability of your survival = $P_8 + P_9 = \frac{1}{12} + \frac{2}{9} = \underline{\underline{\frac{11}{36}}}$

(III) Suppose you shoot at the air. Then it is Cowboy B to begin the game.

Cowboy B must shoot at Cowboy C to maximize his chance of survival.

(a) If Cowboy B hits Cowboy C, then it becomes a "two-way duel" between you and Cowboy B with you shoot first.

Therefore your probability of survival is $P_{10} = \frac{1}{2} \times P_3 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(b) If Cowboy B misses in shooting Cowboy C, then it is Cowboy C's turn and he kills Cowboy B.

What remains is a "two-way duel" between you and Cowboy C with you shoot first.

Therefore your chance of survival is $P_{11} = \frac{1}{2} \times P_1 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\therefore$ Total probability of your survival = $P_{10} + P_{11} = \frac{1}{4} + \frac{1}{6} = \underline{\underline{\frac{5}{12}}}$

As a whole, since $\frac{5}{18} < \frac{11}{36} < \frac{5}{12}$, **(III)** is the best choice, you should shoot in the air!